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BOILING-LIQUID OUTFLOW FROM A FINITE-VOLUME VESSEL THROUGH A SLOT

F. R. Khuzina

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The outflow of a vapor-liquid system from a large vessel through a slot has been considered. For the adiabatic flow of a liquid coming to the boil, a barotropic equation of state is proposed. It is shown that, depending on the conditions inside the vessel and at its outlet, the outflow process proceeds in both the regime of gas-dynamic confinement and the subsonic regime. Examples of numerical calculations are given.

Introduction. The great interest of researchers in the problem and tasks of the mechanics of multiphase media is due to the wide distribution of such systems in nature and their intensive use in modern technology. Experiments on the high-speed outflow through vessels and channels of various diameters that are of interest for analyzing the consequences of a seal failure of high-pressure vessels entail large expenditures. Therefore, the construction of theoretical models that permit describing the outflow process in a wide range of regime parameters is very urgent.

In the present paper, it is shown that in many cases of great interest for practical purposes (evacuation of a vessel filled with saturated propane), the suppressing stage of the outflow process occurs in a more sluggish regime. In so doing, the sluggishness is insignificant and the evacuation rate is determined by the hydraulic loss and the effects of adiabatic expansion of the boiling liquid.

Problem Formulation and Basic Equations. Some results on the wave outflow of a liquid coming to the boil, which is the initial stage of evacuation of channels, neglecting the hydraulic friction forces on the channel walls, as well as some questions of the stationary outflow from a large vessel through a slot are given in [1–3].

Consider a problem on the evacuation of a vessel filled with a boiling liquid through a slot. We assume that the pressure in the main volume at a sufficient distance from the slot is uniform (homobaricity condition [3]) and the outflow process is quasi-stable [3]; thus, the dynamic processes are localized mainly in the flow region adjoining the slot. In so doing, the distribution of parameters in this region is analogous to the distribution in a stationary flow [2]. Let us write the mass equation for the gas-liquid mixture in the volume V:

$$V \frac{d\rho_{\mathrm{i}}}{dt} = - S\rho_{\mathrm{e}} w_{\mathrm{e}} \, . \tag{1}$$

By virtue of the above assumptions, the pressures in the volume and on the outlet cross section are related by the Bernoulli integral

$$\frac{w_{\mathrm{e}}^2}{2} + \int_{p_{\mathrm{e}}}^{p_{\mathrm{i}}} \frac{dp}{\rho} = 0 \, , \tag{2}$$

$$\frac{1}{\rho} = \frac{1 - x}{\rho_{\mathrm{liq}}^0} + \frac{x}{\rho_{\mathrm{g}}^0} \, , \tag{3}$$

where ρ_i^0 (i = liq, g) denotes the true phase densities. Taking into account [4] that $x = c_{\mathrm{liq}}(T_0 - T_{\mathrm{s}}(p))l^{-1}$, we obtain

Birsk State Pedagogical Institute, 10 Internatsional'naya Str., Birsk, 452453, Russia; email: HuzinaFR@yandex.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 78, No. 3, pp. 141–144, May–June, 2005. Original article submitted February 12, 2004.

$$\frac{1}{\rho} = \frac{1}{\rho_{liq}^0} + \left(\frac{1}{\rho_g^0} - \frac{1}{\rho_{liq}^0}\right)\frac{c_{liq}\,(T_0 - T_s\,(p))}{l}\,, \tag{4}$$

$$\rho_g^0 = \frac{p}{RT_s\,(p)}\,.$$

The dependence $T_s(p)$ satisfies the Clausius equation

$$\frac{dT_s\,(p)}{dp} = \frac{T_s}{l}\left(\frac{1}{\rho_g^0} - \frac{1}{\rho_{liq}^0}\right). \tag{5}$$

In view of (5), Eq. (4) transforms to the form

$$\frac{1}{\rho} = \frac{1}{\rho_{liq}^0} + \frac{c_{liq}\,(T_0 - T_s\,(p))\,T_s^{\prime}\,(p)}{T_s\,(p)}\,, \tag{6}$$

where c_{liq} is the specific heat capacity of the liquid and the prime indicates the derivative with respect to pressure. To describe the pressure dependence of the saturation temperature [4], the following expression is used:

$$T_s\,(p) = T_*/\ln\,(p_*/p_0)\,,$$

where T_* and p_* are the empirical parameters. Thus, formula (6) is a barotropic equation of state of a liquid coming to the boil at its adiabatic outflow. On the basis of Eq. (6) one can determine the velocity of sound

$$C^{-2} = \frac{d\rho}{dp} = \rho^2 c_{liq}\,\frac{T_0 T_s^{\prime 2}\,(p) - (T_0 - T_s\,(p))\,T_s\,(p)\,T_s^{\prime\prime}\,(p)}{T_s^2\,(p)}\,. \tag{7}$$

Two outflow regimes are possible. In the first regime, outflow occurs under the conditions of gas-dynamic confinement where the outflow rate w_e is equal to the local velocity of sound. In this case, the value of the pressure p_c at the outlet cut of the slot is larger than the external pressure p_e and is found from the equation

$$C^2\,(p_c) = w^2\,(p_c)\,, \tag{8}$$

and the value of $w(p_c) = w_e$ is determined by means of the Bernoulli integral (2). The second regime is subsonic, where the value of the pressure p_c becomes equal to the external atmospheric pressure. Using relations (2), (6), and (7), from (8) we obtain the transcendental equation

$$\frac{T_s^2\,(p_c)}{\rho^2\,(p_s)\,c_{liq}\,T_0 T_s^{\prime 2}\,(p) - (T_0 - T_s\,(p))\,T_s\,(p)\,T_s^{\prime\prime}\,(p)} = \frac{p_i - p_c}{\rho_{liq}^0} + c_{liq} T_0 \ln\frac{T\,(p_i)}{T\,(p_c)} + c_{liq}\,(T\,(p_c) - T\,(p_i)) \tag{9}$$

for determining p_c depending on the current pressure value inside the vessel p_i. Using the relation

$$\frac{d\rho_i}{dt} = \frac{d\rho_i}{dp_i}\frac{dp_i}{dt} = \frac{1}{C^2\,(p_i)}\frac{dp_i}{dt}\,,$$

from (1) we can get

$$\frac{dp_i}{dt} = D = -\frac{S}{V}\,C^2\,(p_i)\;\rho_e\,(p_c)\,w_e\,(p_c)\,. \tag{10}$$

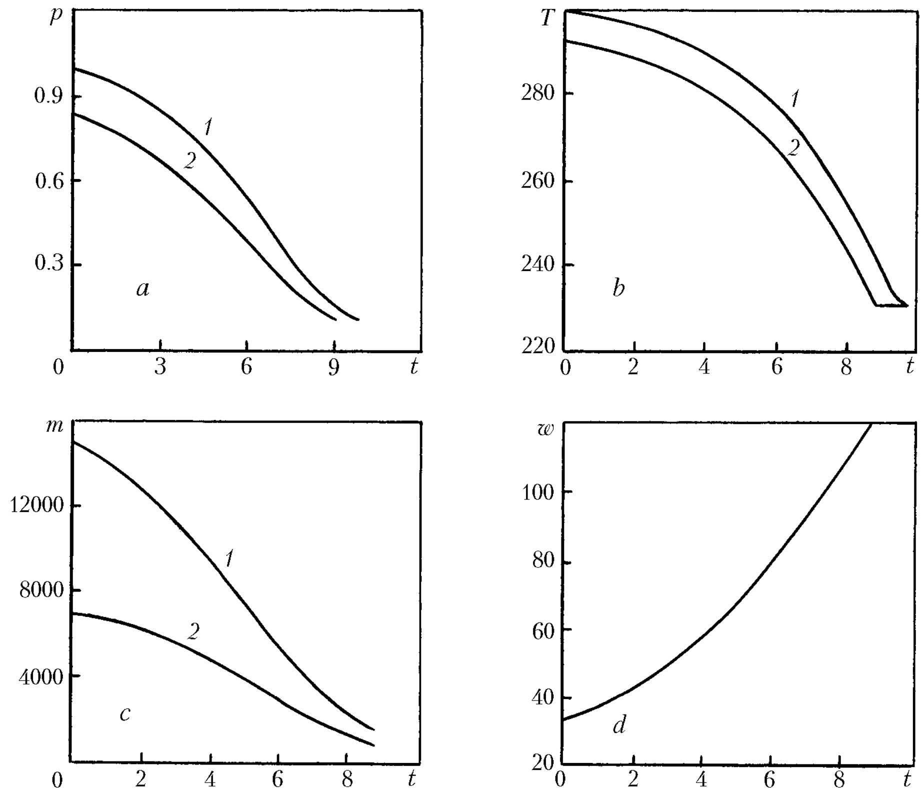

Fig. 1. Change with time in the pressure (a), temperature (b), and mass flow (c) of the mixture inside the vessel (curve 1) and at the slot cut (curve 2); change with time in the outflow velocity (d) of the mixture inside the vessel. p, MPa; T, K; m, kg/(m^2·sec); w, m/sec; t, sec.

Expression (10) describes the change in the pressure inside the vessel with time. It is impossible to obtain an explicit analytical relation between p_i and p_c on the basis of the transcendental equation (9). Therefore, for convenience in performing numerical calculations, let us differentiate (10) with respect to time:

$$\frac{dC^2(p_\mathrm{i})}{dp_\mathrm{i}}\frac{dp_\mathrm{i}}{dt} = \frac{1}{\rho^0_\mathrm{liq}}\left(\frac{dp_\mathrm{i}}{dt}-\frac{dp_\mathrm{c}}{dt}\right) + c_\mathrm{liq}T_0\left(\frac{T'(p_\mathrm{i})}{T(p_\mathrm{i})}\frac{dp_\mathrm{i}}{dt} - \frac{T'(p_\mathrm{c})}{T(p_\mathrm{c})}\frac{dp_\mathrm{c}}{dt}\right) + c_\mathrm{liq}\left(T'(p_\mathrm{c})\frac{dp_\mathrm{c}}{dt} - T'(p_\mathrm{i})\frac{dp_\mathrm{i}}{dt}\right).$$

Then for p_c we will obtain the dependence

$$\frac{dp_\mathrm{c}}{dt} = \frac{\dfrac{dC^2(p_\mathrm{i})}{dp_\mathrm{i}} - \dfrac{1}{\rho^0_\mathrm{liq}} - \dfrac{c_\mathrm{liq}T_0T'(p_\mathrm{i})}{T(p_\mathrm{i})} + c_\mathrm{liq}T'(p_\mathrm{i})}{c_\mathrm{liq}T'(p_\mathrm{c}) - \dfrac{c_\mathrm{liq}T_0T'(p_\mathrm{c})}{T(p_\mathrm{c})} - \dfrac{1}{\rho_\mathrm{liq}}}\,\frac{dp_\mathrm{i}}{dt},$$

which, with regard for the designations

$$A = \frac{dC^2(p_\mathrm{i})}{dp_\mathrm{i}} - \frac{1}{\rho^0_\mathrm{liq}} - c_\mathrm{liq}T_0\frac{T'(p_\mathrm{i})}{T(p_\mathrm{i})} + c_\mathrm{liq}T'(p_\mathrm{i}),\quad B = c_\mathrm{liq}T'(p_\mathrm{c}) - c_\mathrm{liq}T_0\frac{T'(p_\mathrm{c})}{T(p_\mathrm{c})} - \frac{1}{\rho_\mathrm{liq}},$$

will take on the form

$$\frac{dp_c}{dt} = \frac{A}{B} D \,. \tag{11}$$

Equation (11) jointly with (10) can be reduced to the system of two differential equations

$$\frac{dp_i}{dt} = D\,(p_c, p_i)\,, \quad \frac{dp_c}{dt} = \frac{A}{B} D \,. \tag{12}$$

Equation (10) should be used before the time $t = t_c$ at which the pressure p_c will drop to the value of atmospheric pressure p_a. Subsequently, at $t \geq t_c$ in Eq. (10) one should assume that $\rho_e = \rho(p_a)$ and $w_e = w(p_a)$; then the subsonic outflow regime ($w_e << C(p_a)$) will be realized.

This system of equations was solved numerically (by the Runge–Kutta method). Figure 1 shows the current values of the pressure (a), temperature (b), mass flow (c), and outflow velocity (d) of propane inside the vessel at the slot cut in the case of evacuation of a vessel of volume $V = 10^2$ m^3 and slot area $S = 1$ m^2. In the initial state it was assumed that the temperature of liquefied propane was $T_0 = 300$ K, $p_0 = 10^6$ Pa, and the external pressure was equal to the atmospheric pressure $p_a = 0.1$ MPa. The physical parameters of the propane were as follows: $c_{liq} = 2400$ J/(kg·K), $\rho_0 = 490$ kg/m^3, $p_* = 1.9 \cdot 10^9$ Pa, $T_* = 2279$ K, and $R = 153.26$ Pa·m^3/(kg·K).

At the initial stage of evacuation where the outflow velocity is equal to the local velocity of sound, it is seen that the outflow velocity w_e increases with time. This is due to the increase in the velocity of sound for the vapor–liquid mixture with adiabatically decreasing pressure. However, the total mass flow through the slot decreases, since the mean density of the mixture ρ_e decreases more intensively than the outflow velocity at an adiabatic flow of a boiling liquid.

CONCLUSIONS

At the initial stage of evacuation of a vessel with a liquid coming to the boil, where the outflow occurs in the sound-confinement regime, an increase in the outflow velocity is observed (while the vessel pressure decreases with time); there is also a decrease in the total mass flow.

NOTATION

c_{liq}, specific heat capacity of the liquid, J/(kg·K); l, specific heat of vaporization, J/kg; m, mass liquid flow, kg/(m^2·sec); p, pressure, Pa; p_a, atmospheric pressure; p_c, pressure at the slot cut, Pa; p_i, pressure value inside the vessel, Pa; R, gas constants, Pa·m^3/(kg·K); S, cross-section area, m^2; t, time, sec; T, current temperature, K; T_0, initial temperature of the liquid, K; T_s, saturation temperature, K; V, volume, m^3; w_e, outflow velocity at the slot cut, m/sec; x, mass vapor precipitation; ρ, density, kg/m^3; ρ_i, mean density of the mixture in the vessel, kg/m^3; ρ_e, density at the slot cut, kg/m^3. Subscripts: 0, true value of a parameter; e, value of a parameter at the external pressure boundary; *, empirical parameter; s, saturation; i, inside the vessel; c, at the slot cut; liq, liquid; g, gas; a, atmospheric.

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